Math 1B
Quiz 7 Version
Wed Nov 21, 2012 (DUE Mon Nov 26, 2012 @ START OF CLASS)

NAME YOU ASKED TO BE CALLED IN CLASS: _____

SCORE: _____ / 30 POINTS

SHOW ORGANIZED & PROPER WORK (INCLUDING SANITY CHECKS) & SIMPLIFY YOUR FINAL ANSWER TO RECEIVE FULL CREDIT

- Sign below to confirm that the work shown on this quiz is strictly your own work.
- You may have consulted your textbook, your notes and the class handouts, and used your calculator,
- but you did NOT consult other people, websites, software or other outside sources of help.

SIGNATURE: _____

Evaluate $\int m^{-4}(\ln m)^2\,dm$.

SCORE: _____ / 4 PTS

$u=(\ln m)^2 \quad dv=m^{-4}dm$

$du=\frac{2\ln m}{m}dm \quad v=-\frac{1}{3}m^{-3}$

$-\frac{1}{3}m^{-3}(\ln m)^2+\frac{2}{3}\int m^{-4}\ln m\,dm$ (½)

$=-\frac{1}{3}m^{-3}(\ln m)^2+\frac{2}{3}\left[-\frac{1}{3}m^{-3}\ln m+\frac{1}{3}\int m^{-4}dm\right]$ (½)

$u=\ln m \quad dv=m^{-4}dm$

$du=\frac{1}{m}dm \quad v=-\frac{1}{3}m^{-3}$

$=-\frac{1}{3}m^{-3}(\ln m)^2-\frac{2}{9}m^{-3}\ln m-\frac{2}{27}m^{-3}+C$ (1) (1) (½) (½)

★ SEE ALTERNATE SOLUTION AT END OF THIS KEY IF YOU USED METHOD IN MY HANDOUT

Evaluate $\int \sin^9 r\cos^5 r\,dr$.

SCORE: _____ / 4 PTS

$=\int \sin^9 r\cos^4 r\cos r\,dr$

$u=\sin r$

$du=\cos r\,dr$

$=\int u^9(1-u^2)^2\,du$ (½)

$=\int (u^9-2u^{11}+u^{13})\,du$

$=\frac{1}{10}u^{10}-\frac{1}{6}u^{12}+\frac{1}{14}u^{14}+C$ (1)

$=\frac{1}{10}\sin^{10}r-\frac{1}{6}\sin^{12}r+\frac{1}{14}\sin^{14}r+C$ (1) (½)

Evaluate $\int \frac{8x^2+11x-6}{x^3+2x^2-4x-8}\,dx$.

$=\int \frac{8x^2+11x-6}{(x+2)^2(x-2)}\,dx$

SCORE: _____ / 5 PTS

SANITY CHECK

$x=3$

$\frac{8x^2+11x-6}{x^3+2x^2-4x-8}=\frac{99}{25}$ (¼)

$\frac{5}{x+2}-\frac{1}{(x+2)^2}+\frac{3}{x-2}$

$=1-\frac{1}{25}+3$ (¼)

$=\frac{99}{25}$ ✓

$=\int\left(\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{x-2}\right)dx$

$=\int\left(\frac{5}{x+2}-\frac{1}{(x+2)^2}+\frac{3}{x-2}\right)dx$ (½) (½) (½)

$=5\ln|x+2|+\frac{1}{x+2}+3\ln|x-2|+C$ (½) (½) (½) (½)

(1)

$8x^2+11x-6=A(x+2)(x-2)+B(x-2)+C(x+2)^2$

$x=2: \ 48=16C \rightarrow C=3$

$x=-2: \ 4=-4B \rightarrow B=-1$

COEF OF x^2: $8=A+C$

$8=A+3 \rightarrow A=5$

Evaluate $\int x^2\sqrt{x^2-9}\,dx$.

$x = 3\sec\theta$ (1)

$dx = 3\sec\theta\tan\theta\,d\theta$

$\sec\theta = \frac{x}{3}$

$= \int 9\sec^2\theta \cdot 3\tan\theta \cdot 3\sec\theta\tan\theta\,d\theta$

$= (1)\ 81\int \sec^3\theta\tan^2\theta\,d\theta$

$= (1)\ 81\int (\sec^5\theta - \sec^3\theta)\,d\theta$

OK IF SIMPLIFIED BY ADDING

$= (1)\ 81\left[\frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{4}\int\sec^3\theta\,d\theta - \int\sec^3\theta\,d\theta\right]$

$= 81\left[\frac{1}{4}\sec^2\theta\tan\theta - \frac{1}{4}\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|\right)\right]$ (½) $+C$

$= (\frac{1}{2})\ \frac{81}{4}\sec^3\theta\tan\theta - \frac{81}{8}\sec\theta\tan\theta - \frac{81}{8}\ln|\sec\theta+\tan\theta| + C$

$= \frac{81}{4}\frac{x^3}{27}\frac{\sqrt{x^2-9}}{3} - \frac{81}{8}\frac{x}{3}\frac{\sqrt{x^2-9}}{3} - \frac{81}{8}\ln\left|\frac{x}{3}+\frac{\sqrt{x^2-9}}{3}\right| + C$

$= (1\frac{1}{2})\ \frac{1}{4}x^3\sqrt{x^2-9} - \frac{9}{8}x\sqrt{x^2-9} - \frac{81}{8}\ln\left|x+\sqrt{x^2-9}\right| + C$ (½)

SCORE: ____ / 6 PTS

Evaluate $\int \frac{\sin 3t}{e^{2t}}\,dt$.

u	dv
$\sin 3t$	$+e^{-2t}$
$3\cos 3t$	$-\frac{1}{2}e^{-2t}$
$-9\sin 3t$	$\frac{1}{4}e^{-2t}$

$= (1)\ -\frac{1}{2}e^{-2t}\sin 3t - \frac{3}{4}e^{-2t}\cos 3t$ (1)

$-\frac{9}{4}\int e^{-2t}\sin 3t\,dt$ (1)

$\frac{13}{4}\int e^{-2t}\sin 3t\,dt = -\frac{1}{2}e^{-2t}\sin 3t - \frac{3}{4}e^{-2t}\cos 3t$ (½)

$\int e^{-2t}\sin 3t\,dt = -\frac{2}{13}e^{-2t}\sin 3t - \frac{3}{13}e^{-2t}\cos 3t + C$

(1) (½)

SCORE: ____ / 5 PTS

Evaluate $\int \frac{3x^2+26}{x^3+4x^2+13x}\,dx$.

$= \int \frac{3x^2+26}{x(x^2+4x+13)}\,dx$

$= \int \left(\frac{A}{x} + \frac{B(2x+4)+C}{x^2+4x+13}\right)dx$

$= \int \left(\frac{2}{x} + \frac{\frac{1}{2}(2x+4)-10}{(x+2)^2+9}\right)dx$ (½) (1)

$= (\frac{1}{2})\ 2\ln|x| + \frac{1}{2}\ln(x^2+4x+13)$ (½)

$-\frac{10}{3}\tan^{-1}\frac{x+2}{3} + C$

(1) (½)

(1½)

$3x^2+26 = A(x^2+4x+13) + Bx(2x+4) + Cx$

$x=0$: $26 = 13A \rightarrow A = 2$

$x=-2$: $38 = 9A - 2C$

$38 = 18 - 2C \rightarrow C = -10$

COEF OF x^2: $3 = A + 2B$

$3 = 2 + 2B \rightarrow B = \frac{1}{2}$

SANITY CHECK

$x = 2$

$\frac{3x^2+26}{x^3+4x^2+13x} = \frac{38}{50} = \frac{19}{25}$ (¼)

$\frac{2}{x} + \frac{\frac{1}{2}(2x+4)-10}{x^2+4x+13}$

$= 1 - \frac{6}{25}$ (¼)

$= \frac{19}{25}$ ✓

SCORE: ____ / 6 PTS

ALTERNATE METHOD FOR $\int m^{-4}(\ln m)^2\,dm$

u	dv
$(\ln m)^2$	m^{-4}
$\frac{2\ln m}{m}$	$-\frac{1}{3}m^{-3}$

$* m$ — — — — — — $* m^{-1}$

$2\ln m$ $\quad -\frac{1}{3}m^{-4}$ $\quad \frac{1}{2}$

$\frac{2}{m}$ $\quad \frac{1}{9}m^{-3}$

$* m$ — — — — — — $* m^{-1}$

2 $\quad \frac{1}{9}m^{-4}$ $\quad \frac{1}{2}$

0 $\quad -\frac{1}{27}m^{-3}$

$$-\frac{1}{3}m^{-3}(\ln m)^2 - \frac{2}{9}m^{-3}\ln m - \frac{2}{27}m^{-3} + C$$

(1) (1) ($\frac{1}{2}$) ($\frac{1}{2}$)